Introduction to the Special Issue on Wavelet Transforms and Multiresolution Signal Analysis

THE analysis of signals and phenomena at multiple scales of resolution is not a new topic. The level of attention and interest it has received in the past few years, however, does represent something new. While we have no intention of providing a detailed commentary on the sociology of science and its particular manifestations in the area of inquiry on which this special issue focuses, we feel that it is worthwhile to make a few comments in order to describe the context in which the research described in this issue has been performed and in which it should be viewed.

Without question the catalyst for much of this comparative frenzy of research activity has been the development of the wavelet transform, which has provided not only a wealth of new mathematical results, but also a common language and rallying call for researchers in a remarkably wide variety of fields: mathematicians working in harmonic analysis because of the special properties of wavelet bases; mathematical physicists because of the implications for time-frequency or phase-space analysis and relationships to concepts of renormalization; digital signal processors because of connections with multirate filtering, quadrature mirror filters, and subband coding; image processors because of applications in pyramidal image representation and compression; researchers in computer vision who have used "scale-space" methods for some time; researchers in stochastic processes interested in self-similar processes, $1/f$ noise, and fractals; speech processors interested in efficient representation, event extraction, and mimicking the human auditory system. And the list goes on.

Thus, at the very least what wavelets have done is to bring a diverse array of researchers together and to provide a considerable impulse of energy to the field. At this point in time, the system is certainly still ringing, and thus, it is far too premature for any of us to have a clear and complete picture of what wavelets have added to our understanding of multiresolution methods—or more fundamentally to have distilled a concise picture of the principles of multiresolution signal analysis and the place held in these by wavelet transforms. Indeed, we are all (including the mathematician, researcher in image processing, and signals and systems specialist who comprise the guest editorial board) still working hard to understand each other and to keep up with the research explosion.

So that tells you one thing that this special issue is not: it is *not* a tour through a more-or-less mature field. Furthermore, the issue does *not* include a tutorial on wavelets, since this young field already has enough good ones for which the ink is barely dry.[1] Indeed, for the same reason we made a conscious decision *not* to have any invited papers (although we did encourage a few people to contribute): rather, we hoped to be able to present a broad cross-section of current research directions that together would provide considerably more than an inkling into the richness and promise of this fascinating topic. You, the reader, can, of course, make your own judgment as to whether or not we and the authors of these papers have succeeded, although we do believe that there are many stimulating ideas presented herein. For us, however, there are two other, albeit indirect, indicators of success. The first is that the collection of authors for this issue includes individuals from an exceptionally wide range of disciplines, providing all of us with a diverse collection of perspectives on wavelets and multiresolution signal analysis. The second is that this diversity extends as well to the topics covered, providing you with a picture of the richness of the field and us with the headache of trying to figure out how to organize the issue.

The first group of papers and correspondences in this issue deal with various aspects and properties of wavelet transforms themselves. Indeed, given the relative youth of the subject, there is still much to be learned about wavelets, and the papers presented here provide a look at how this investigation is progressing. The paper by Kovačević and Vetterli studies the construction and properties of multidimensional wavelet bases that are not given by simple tensor products of one-dimensional schemes. An example of such a construction is given by quincunx-subsampling for images, which has the advantages of treating verticals, horizontals, and diagonals on the same footing. Gröchenig and Madych also concentrate on nonseparable multidimensional schemes. They restrict themselves to the case where the scaling function takes only the two values 0 and 1; the corresponding wavelet bases are multidimensional generalizations of the Haar basis. They show that many surprising self-similar sets which tile the plane arise naturally. Rioul and Duhamel provide a tutorial review of fast wavelet transform algorithms and develop guidelines for implementing discrete and continuous wavelet transforms efficiently. They focus on the computational structures of the algorithms rather than on the mathematical relationship between transforms and algorithms. That the discrete wavelet transform is not translation-invariant is a crucial problem for pattern recognition applications. Simon-

[1] Although several papers contain brief summaries of wavelet transform properties, so that reading this special issue does not require extensive previous knowledge about wavelets.

celli, Freeman, Adelson, and Heeger introduce an interpolation procedure that recovers translation invariance. They extend this idea to dilation- and rotation-invariance and describe applications to stereo-matching and image enhancement. The paper by Munch does not directly concern wavelets. He gives a mathematical proof of the increased accuracy that can be obtained when using windowed Fourier transforms at a higher than Nyquist density. In practice, this means that a signal can still be reconstructed with high precision, even if the coefficients are not very precise. This had been observed for both the wavelet and the windowed Fourier transform, and only partially explained; the editors hope that Munch's paper will stimulate work on the wavelet equivalent. The correspondence by Olsen and Seip computes frame bounds, which determine the convergence of reconstruction algorithms, for more general families of one-dimensional wavelets than those generated by a regular lattice of dilations and translations. Unser, Aldroubi and Eden describe the properties of B-spline wavelets. They show that they have an optimal joint resolution in the spatial and Fourier domains, in the sense that they converge to modulated Gaussians when the order of the B-spline tends to infinity. Volkmer gives in his correspondence a simple and elegant argument to establish the exact asymptotic regularity of the orthonormal bases of compact support and maximal number of vanishing moments for their support width. Colella and Heil discuss the structure and properties of all the orthonormal wavelet bases that correspond to subband filters with four taps. Walter extends the standard sampling theorem to multiresolution spaces. Janssen shows that the scaling function in a nontrivial orthonormal wavelet basis necessarily takes on negative values.

The second group of papers deals with various approaches and problem formulations associated with the isolation of events in time (or space) and, perhaps, frequency. This is in fact one of the promised advantages of wavelet transforms, and the papers and correspondences in this group explore and develop this promise. Mallat and Hwang give a tutorial overview of the characterization of singularities with the wavelet transform. They detect and analyze singularities from the local maxima of the wavelet transform modulus. An application to noise removal in one- and two-dimensional signals is described. The paper by Delprat, Escudié, Guillemain, Kronland-Martinet, Tchamitchian, and Torrésani uses stationary phase approximation, a technique dear to physicists, to extract characteristic "signatures" from the wavelet transform or the windowed Fourier transform of a signal. They give examples, which include the determination of frequency and amplitude modulation laws. In their paper, Friedlander and Porat focus on a more classical detection problem, namely the detection of transient waveforms from noisy received signals, using wavelet and other data transformations as candidate preprocessing stages to focus information for transient localization and detection. The principal aim of the authors is to investigate the robustness of several such detection schemes to the inevitable mismatch between actual transient events and the models for them used in detector design. Wilson, Calway, and Pearson define an extension of the wavelet transform and of the windowed Fourier transform that contains aspects of both; they present applications to acoustic and image analysis. Bovik, Gopal, Emmoth, and Restrepo study the detection and measurement of local emergent "frequencies" in signals. Local frequency components are measured from the wavelet coefficients computed with Gabor wavelets. They apply this technique to texture characterization. In the first correspondence in this group, White addresses problems related to the synthesis of wideband signal waveforms and the recovery of signals from wide-band ambiguity functions. In the second correspondence, Frisch and Messer use the wavelet transform to detect unknown transients in a signal.

The third group of papers and correspondences deals with the use of wavelet transforms and their natural extension to wavelet packets to design efficient schemes for representing and compressing signals and images. This is an extremely active area of research, and the papers here describe several different lines of investigation. The paper by Coifman and Wickerhauser gives a summary of their "wavelet packet" method, as well as a technique using lapped window transforms, together with an algorithm that searches a dyadic tree for the best possible basis. They illustrate their method with a speech example. DeVore, Jawerth, and Lucier formalize the image compression problem in a functional analysis setting. They analyze the errors produced by the quantization of the wavelet transform coefficients and explain how to match the error measure to human visual sensitivity. Examples of image compression are shown. The paper by Tewfik, Sinha, and Jorgensen deals with the problem of adapting the choice of the wavelet itself in order to achieve the best scale-limited approximation of a given signal. A significant aspect of this work is the choice of cost criteria that yield both good results and efficients algorithms. In their correspondence, Delsarte, Macq, and Slock describe an algorithm that adapts the filters that implement the wavelet transform in order to maximize the coding gain for each signal. They compare their compact image coding results with the performance of the discrete cosine transform.

Multiresolution models and methods of analysis for fractals and stochastic phenomena is the focus of the fourth group of papers. The notion of self-similarity arising in both the theory of stochastic processes and in the description of fractals would seem to be particularly well-suited to multiscale methods, and the papers in this group explore this and several other equally important concepts. The first paper by Basseville, Benveniste, Chou, Golden, Nikoukhah, and Willsky explores the multiscale modeling of stochastic processes using scale-recursive stochastic models whose pyramidal structure is similar to that found in wavelet transforms and other multiscale signal representations. The models studied in this paper lead to extremely efficient algorithms for scale-recursive model-building and optimal estimation, generalizing the Levinson and Kalman filtering algorithms. In their paper, Wornell and Oppenheim also consider the construction of signals, in this case making explicit use of the wavelet transform. In particular, they construct a class of deterministically self-similar signals whose fractal structure

make them interesting candidates for secure communication through "fractal modulation." The third paper, by Moulin, O'Sullivan, and Snyder focuses on one particular aspect of multiresolution modeling and estimation, namely the flexibility it provides in dealing with noise/resolution trade-offs. In particular these authors propose the use of a multiresolution sieve based on B-splines for the regularization of maximum-likelihood estimation problems, such as arise in radar imaging and spectrum estimation, and use this to derive an asymptotic expression for the optimal resolution. In the final paper in this group, Barbé proposes and analyzes a multiresolution method for studying the level-crossing behavior of Gaussian processes. This approach, based on sampling and linear interpolation, leads to a well-defined, computable quantity for processes such as fractional Brownian motion, (fBm), and in fact provides a method for determining the fractal dimension of such processes. The two correspondence items also deal with the class of fractional Brownian motions, focusing on the use of wavelet transforms in their analysis. Tewfik and Kim examine the correlation structure of the wavelet coefficients of fBm's and show that the rate of inter- and intra-scale decay in correlation depends strongly on the number of vanishing moments in the analyzing wavelet. Finally, Flandrin collects together a number of facts concerning wavelets and fBm's including second-order properties of the wavelet coefficients, methods for estimating fractal dimension from the variance structure of these coefficients, and the synthesis of fBm-like processes using the midpoint displacement method.

While various applications of multiresolution and wavelet-based methods are used in many of the preceding papers to motivate or illustrate formulations and techniques, we have reserved for the fifth and final group papers for which particular applications are the focus. Yang, Wang, and Shamma present an analytical framework to describe mechanical and neural processing in the early stage of the auditory system. They relate the wavelet transform to auditory processing and discuss its applications to automatic speech recognition and data compression. The paper by Healy and Weaver treats applications of wavelets to magnetic resonance imaging. They sketch an approach by which certain images can be acquired faster, using wavelets, than with other currently used methods, and they discuss a technique for "instant" imaging avoiding fast pulsed gradients. The correspondence by Kadambe and Boudreaux-Bartels describes a specific application of the wavelet transform to pitch detection in speech signals. The authors give numerical comparisons with classical pitch detectors in presence of noise.

As we have said, wavelet transform and multiresolution signal analysis is an extremely rich and dynamic area of research touching and bringing together researchers in an exceptionally diverse array of disciplines. We hope that the papers contained in this issue provide you with a panorama not only of the results and accomplishments within this area but also of the tremendous promise that this infant field holds. While this special issue is not the first word on the subject, it certainly is far from the last.

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