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ECG/VCG Rhythm Diagnosis Using Statistical Signal Analysis—II. Identification of Transient Rhythms

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Abstract-The problem of detection and identification of cardiac transient rhythms, using the associated R-R interval sequence, is studied. A generalized likelihood ratio technique is proposed, in which the transient rhythm category is identified by means of a maximum-likelihood hypothesis test. Simultaneously, the magnitude of the change in the R-R interval pattern is estimated. The method is easily mechanized on-line using a moving window of data and prestored gains. Experimental results using actual data are presented to indicate the utility of the method.

I. INTRODUCTION

THIS PAPER is the second of a two-part series on the development of an automated technique for cardiac arrhythmia detection and identification. In Part I [1] the motivation and background for this study were given, and a multiple model technique was developed for detection and identification of persistent rhythms, i.e., rhythms which are essentially unchanged over approximately 8-10 heartbeats.

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M. C. Lancaster and J. H. Triebwasser are with the U.S. Air Force School of Aerospace Medicine, Brooks Air Force Base, TX 78235. In addition, we presented results that showed that, with the aid of an outlier test, the multiple model algorithm was capable of detecting and adapting to switches between persistent rhythm patterns. Although this method does allow one to detect certain sudden changes in a rhythm pattern, its simplicity does not allow one to correctly identify many ectopic events such as compensatory prematures and interpolated beats.

In this paper we investigate the use of a Generalized Likelihood Ratio (GRL) technique [2-4] for detection and identification of transient arrhythmias; e.g., arrhythmias that persist over less than 8-10 heartbeats. The GLR approach is a practical method for detecting and classifying several types of transient events and for estimating the parameters that characterize the events (e.g., the degree of prematurity of a PVC). This approach has previously been found to give good experimental results [3] and will be shown to give excellent results in this application.

Following the methodology in Part I, our approach to modeling is phenomenological in nature; that is, the models are based on rather simple observations concerning the distinguishing characteristics of the R-R interval patterns corresponding to the various transient events. This produces a simple and quite reliable statistical identification procedure. The diagnostic capabilities of this technique are limited only

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by the amount of information contained in the observed pattern of R-R intervals. Thus, the technique we propose can detect all arrhythmias that cause some type of aberrancy in this pattern, (which includes most arrhythmias), but it cannot distinguish between those that cause the same type of disturbance. The point of view taken here for transient arrhythmia analysis is that the R-R interval pattern is a transient variation from an underlying regular rhythm pattern. This appears to be the way in which these patterns are viewed by the cardiologist. Taking this point of view, we will see that the GLR technique operates by performing statistical tests on the measurement innovations produced by a filter based on the "small variation" rhythm model described in Part I.

II. CHARACTERIZATION OF AND MODELS FOR TRANSIENT ARRHYTHMIAS

Let $y(1), y(2), \dots, y(N)$ be a sequence of observed R-R intervals. As described in [1], if the y's come from a regular rhythm containing only small random variations about the mean R-R interval, an appropriate sequential phenomenological scalar model is

$$x(k) = x(k-1), \quad y(k) = x(k) + v(k) \tag{2.1}$$

where x is the ideal, perfectly constant sequence of R-R intervals, and v(k) is a sequence of zero-mean, uncorrelated random variables with variance R. If the sequence of R-R intervals contains an ectopic event, the model (2.1) must be modified. In the rest of this section we describe a number of such events and develop the modifications to (2.1).

Rhythm Jump: This class is characterized by a sudden change of the heart rate, which occurs in the case of onset of bradycardia or tachycardia. The model for this is

$$x(k) = x(k-1) + \nu \delta_{\theta,k}.$$
 (2.2)

Here, ν is the unknown size of the shift in the average *R*-*R* interval at the unknown time, θ , of the shift of the rhythm and δ_{ij} is the Kronecker delta ($\delta_{ii} = 1$; $\delta_{ij} = 0$, $i \neq j$). Thus $\nu > 0$ models bradycardia onset and $\nu < 0$ models tachycardia onset.

Non-Compensatory Beat: This class is characterized by intermittent premature QRS complexes, in which there is incomplete compensation of the R-R interval subsequent to the premature beat, or by dropped QRS complexes in which a much longer than normal R-R interval results. This class includes sinus arrest, SA block and atrial prematures.

For this class of ectopic events, there is either a shortened or lengthened R-R interval, followed by a return to the regular pattern. Thus:

$$x(k) = x(k-1) + \nu [\delta_{\theta,k} - \delta_{\theta,k-1}].$$
(2.3)

Here $\nu < 0$ models a premature beat, while $\nu > 0$ can be used to model a skipped beat.

Compensatory Beat: This class of arrhythmias is characterized by intermittent premature QRS complexes in which complete compensation of the R-R interval is achieved subsequent to the premature beat. Thus, the interval between the QRS complex preceeding the premature and the post-premature QRS complex is equal to two normal R-R intervals. This class includes AV nodal prematures and ventricular prematures. The model for this is:

3

$$\kappa(k) = \kappa(k-1) + \nu [\delta_{\theta,k} - 2\delta_{\theta,k-1} + \delta_{\theta,k-2}].$$
 (2.4)

Double Non-Compensatory Beat: This arrhythmia class is characterized by one of the following patterns: (1) an underlying uniform R-R interval upon which is superposed intermittent extra or ectopic beats called interpolated beats (these beats do not interfere with the normal ventricular rhythm), (2) a double premature, or (3) the dropping of alternate beats (2:1 block). For this case, we are seeking a model which is characterized by two successive lengthened or shortened R-Rintervals. The model for this is

$$x(k) = x(k-1) + \nu [\delta_{\theta,k} - \delta_{\theta,k-2}].$$
(2.5)

III. THE GENERALIZED LIKELIHOOD RATIO (GLR) Algorithm

In this section we develop the necessary equations that define the algorithm for the detection and classification of what we have termed "transient arrhythmias." Recall from the previous section that each of the arrhythmia models developed in Section II is of the form

$$x(k) = x(k-1) + F_i(k,\theta) v$$
(3.1)

$$y(k) = x(k) + v(k)$$
 (3.2)

where F_i takes on the forms implied in Equations (2.2)-(2.5). Here the term $F_i(k, \theta) \nu$ represents an abrupt system change of type *i*. The purpose of the GLR method is to determine if such a change has occurred, and to ascertain the type *i* of the change, the time θ of its occurrence, and its magnitude ν .

Suppose we design a Kalman filter for the "small variation" persistent rhythm model (see Part I) based on the assumption that there are no abrupt changes. The filter is defined by the equations

$$\widehat{x}(k) = \widehat{x}(k-1) + M(k)\gamma(k)$$
(3.3)

$$\gamma(k) = y(k) - \hat{x}(k-1)$$
 (3.4)

where we have the precomputable equations

$$V(k) = P(k-1) + R$$
(3.5)

$$M(k) = P(k-1)/V(k)$$
(3.6)

$$P(k) = [1 - M(k)] P(k - 1).$$
(3.7)

Here we must initialize the equations with the a priori statistics of x(0)-i.e., values for the mean $\hat{x}(0)$ and the covariance P(0). Also under the hypothesis of no abrupt change, the measurement innovations $\gamma(k)$ is a zero-mean, uncorrelated process with covariance V(k). As in the multiple model approach, the GLR method relies on the examination of these innovations.

Suppose now that an abrupt change $F_i(k, \theta) \nu$ occurs. Straightforward calculations [3] show that the innovations can then be written in the form

$$\gamma(k) = G_i(k,\theta) \nu + \tilde{\gamma}(k)$$
(3.8)

where $\tilde{\gamma}$ is the innovations when there is no abrupt change, and the term $G_i(k, \theta) \nu$ reflects the effect of the abrupt change on the innovations. The matrix $G_i(k, \theta)$ is defined by the equations

$$G_{i}(k,\theta) = [1 - M(k - 1)] G_{i}(k - 1,\theta) + F_{i}(k,\theta)$$
(3.9)

$$G_i(\theta, \theta) = F_i(\theta, \theta). \tag{3.10}$$

(Note that F and G are all zero for $k < \theta$.)

The GLR method consists of the following two steps:

(a) Assuming each type of change has occurred, compute the most likely time of its occurrence and the most likely magnitude.

(b) Given these maximum likelihood estimates, we choose the most likely type and decide if it is likely enough to have actually occurred.

The details of this approach (for a more general model than (3.1), (3.2)) are discussed in [5], and we limit ourselves here to the presentation of the relevant equations. Define the (precomputable) quantity

$$C_i(k,\theta) = \sum_{j=\theta}^k G_i^2(j,\theta)/V(j).$$
(3.11)

We can then find an expression for the best estimate of ν as a function of assumed values of θ and *i* and of the observed innovations $\gamma(1), \dots, \gamma(k)$

$$\hat{\nu}(k,\theta,i) = d_i(k,\theta)/C_i(k,\theta)$$
(3.12)

$$d_i(k,\theta) = \sum_{j=\theta}^k G_i(j,\theta) \gamma(j) / V(j).$$
(3.13)

We also compute the log-likelihood ratios

$$l(k,\theta,i) = d_i^2(k,\theta)/C_i(k,\theta)$$
(3.14)

and compute the best estimates $\hat{i}(k)$, $\hat{\theta}(k)$ as those values that maximize $l(k, \theta, i)$. The estimate $\hat{\nu}(k)$ is the corresponding $\hat{\nu}(k, \theta, i)$.

The fundamental quantity used in the GLR test is the transient rhythm "Signature" $G_i(k, \theta)$. The signature for a given transient event describes the systematic behavior of the prediction errors of the Kalman filter when the transient event takes place, and the log-likelihood ratio $l(k, \theta, i)$ is a direct measure of how closely the actual prediction errors resemble the signature $G_i(k, \theta)$. For example, in the case of a rhythm jump, the Kalman filter predicts the next R-R interval based on the old heart rate, and one would expect to observe a consistent deviation between the predicted and the actual R-R intervals following the jump. In this case $l(k, \theta, i)$ will reflect the presence or absence of this deviation. Of course, since the filter has nonzero gain, it will eventually adjust to the new heart rate, and thus the mean prediction error will decay to zero. Clearly the rate at which this decay occurs depends on the filter gain-the larger the gain, the faster the decay. Here we see a potential tradeoff in the system design: one wants enough filter gain in order for the filter to be able to track slowly changing heart rates, but a large gain leads to fast decay of the signatures which in turn makes event detection more difficult.

Figure 1 depicts the GLR algorithm for the rhythm models

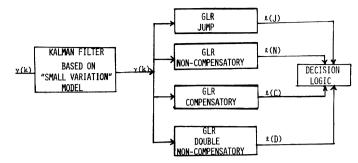


Fig. 1. Transient Rhythm Detection Configuration.

described in Section II. The residuals from the Kalman filter are fed into four systems, one looking for each type of transient event. Each GLR system computes a log-likelihood ratio (l(J), l(N), l(C), l(D)) which measures the likelihood that the particular arrhythmia actually occurred. Decision logic then chooses the most likely arrhythmia type and decides if it is sufficiently likely to have actually occurred. The decision rule for the GLR system is of the form

$$\max \{l(J), l(N), l(C), l(D)\} \stackrel{\text{declare detection}}{\leftarrow} \epsilon \qquad (3.15)$$

NO DETECTION

The choice of ϵ is determined by consideration of the problems of false positives and missed detections. The results presented in the next section indicate that $\epsilon = 15$ is an appropriate value.

We note that GLR in principle requires a growing memory i.e., at time k we must consider all possible values of $\theta \in \{1, \dots, k\}$. A practical and reasonable method for avoiding this difficulty is to consider a "sliding window." At any time k we only consider the last N values of θ

$$\theta \in \{k - N + 1, \cdots, k\}.$$

The idea here is that any change occurring more than N time units in the past would have been detected already. The size N must be chosen with care. If it is taken to be large, we are able to collect a great deal of information and can make a decision with more certainty. However, the larger N is, the larger the computational load. Also, if a particular R-R interval sequence contains several transient events, we want to keep Nsmall to minimize the confusion in separating the various events. In connection with this, we note that GLR is directed at detecting a single transient event. However, as we will see, the technique can be used successfully to detect several events without adjusting the filter subsequent to the detection of an event. There are methods for adjusting the filter after event detection, and we refer the reader to [3, 5] for details.

One difficulty with the GLR method as described so far is the detection of transient events that occur at the start of a record. The GLR filter, which is trying to estimate the average R-R interval, initially has no data on which to base its estimate. Thus, the filter tends to "follow" the first few intervals, and the GLR detector, which is looking at the filter behavior in order to determine if a transient event has occurred, will be fooled. Of course, as we smooth the data by processing more and more data points, the GLR will, in prin-

N number of beats	Kalman Filter Parameters			
	P(0) = 32		P(0) = 1600	
	Gain M(i)	Innovations Variance V(i)	Gain M(i)	Innovations Variance V(i)
1	0.333	96.0	0.962	1664.0
2	0.250	85.3	0.490	125.5
3	0.200	80.0	0.329	95.4
4	0.167	76.8	0.248	85.1
5	0.143	74.7	0.198	79.8
6	0.125	73.1	0.166	76.7
7	0.111	72.0	0.142	74.6
8	0.100	71.1	0.124	73.1
9	0.100	70.4	0.111	72.0
10	0.100	69.8	0.100	71.1
11	0.100	69.3	0.100	70.4
12	0.100	68.9	0.100	69.8
13	0.100	68.6	0.100	69.3
14	0.100	68.3	0.100	68.9
15	0.100	68.0	0.100	68.6

TABLE I "Small Variation" Filter Parameters

ciple, be able to determine that it is the first few beats that contain the problem, but for short record lengths or for narrow GLR windows, one may not be able to obtain enough smoothing in this manner. Motivated by the observed problem of one aberrant interval causing an error in computing an initial average, the following initialization scheme was used:

Step (1): Search the first 5 beats and find the first two consecutive intervals y(k) and y(k + 1) that satisfy

$$|y(k) - y(k+1)| < \beta.$$

Step (2): Set the initial filter estimate equal to their average

$$\hat{x}(0) = \frac{y(k) + y(k+1)}{2}$$

and set the initial covariance P(0) to $\frac{1}{2}$ of the noise covariance associated with the measurement of a normal *R-R* interval. This reflects accurately the variance associated with an estimate of a random variable obtained by averaging two samples.

Step (3): If none of the first 5 beats is less than β apart, the initial filter estimate is set equal to the average of the first 5 beats, and P(0) is set to a lower value.

For the actual runs described in the next section, we have taken $\beta = 20$ and P(0) = 32.¹

IV. EXPERIMENTAL RESULTS

The GLR detection system presented in Section III has been implemented and tested on a variety of idealized and actual *R-R* interval data. All *R-R* intervals were determined using the *R*-wave detector documented in [5]. The idealized data were used to evaluate performance under ideal conditions when the transients followed one of the models exactly. Our results consist of plots of the log-likelihood ratios. The performance of the decision rule (3.15) can be directly assessed from these plots by choosing a value of ϵ . We have found that $\epsilon = 15$ yields good results.

A summary of the parameters of the "small variation"

Kalman filter used to generate the *R*-*R* interval estimates under the "no transient" hypothesis is given in Table I. Shown are the values of filter gain for two values of initial error covariance; P(0) = 32 and P(0) = 1600, in units of (samples)², at 4 ms/sample. Note that the gain decreases monotonically to M(k) = 0.100 and is held at that value thereafter in order to keep the filter active; i.e., so that the "small variation" filter can track a slowly varying rhythm. Also shown in the table are the corresponding innovations variances, which are used in the GLR tests.

The first test made consisted of a pure jump. The data were obtained by piecing together two actual normal sinus rhythm strips with different heartbeat rates. The results are shown in Figures 2-6. In the figures the likelihoods $L(k, \theta)^2$ are plotted vs. θ for constant k. The vertical marks along the abscissa represent the actual time at which the R-wave occurred. The numbers along the abscissa represent the number of 4 ms samples and seconds in each heartbeat. The values for k < 6have been suppressed for convenience. Note that L(k, 6) increases monotonically with k for k > 6 in Figure 3, a strong indication that a jump took place at $\theta = 6$. The likelihoods for the other three categories considered are shown in Figures 3-5 and show no strong monotonicity at any point. From this, it is concluded that a jump occurred at $\theta = 6$. The jump estimate in Figure 2 is seen to be quite accurate. The results in Figures 2-5 may be summarized, in this case, by plotting $\max_{\theta} L(k, \theta)$ vs. k for each transient event class. The results are shown in Figure 6. Note that the jump is identified at k = 8 (i.e., it is distinguished from the other possible arrhythmias). It cannot be identified at k = 7 since the "double non-compensatory" category is equivalent to a jump of length two. Note that all transient rhythm categories are equally likely at k = 6, since only a single "outlier" (event of length one) has been detected.

The next test was performed on an actual rhythm strip in which there was gradual slowing to sinus bradycardia followed by slow nodal rhythm. A summary plot is shown in Figure 7.

¹As discussed in Part I, the unit in which *R*-*R* intervals is expressed is 4 ms. Thus, for example, $\beta = 80$ ms.

 $^{{}^{2}}L(k,\theta)$ for class *i* is the statistical log-likelihood ratio $l(k,\theta,i)$ of Eq. (3.14).

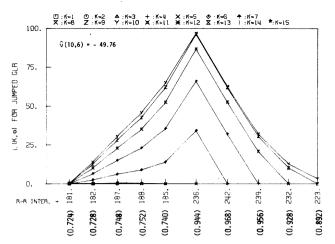


Fig. 2. Jump Likelihood for Rhythm Jump. L(k, 6) increases monotonically with k indicating increasing confidence in jump at $\theta = 6$. $\hat{v}(10, 6)$ is the jump estimate for the sixth interval after ten intervals have been processed.

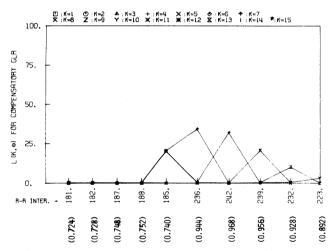


Fig. 4. Compensatory Likelihood for Rhythm Jump. L(k, 6) decreases monotonically indicating that compensatory beat did not occur.

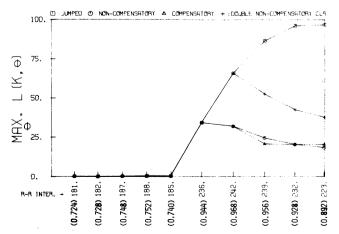


Fig. 6. Maximum Likelihoods for Rhythm Jump. L(6, 6) = 35 for all classes. L(7, 6) is reduced for compensatory and non-compensatory classes, but increased for double non-compensatory and jump since they are not distinguishable until k = 8.

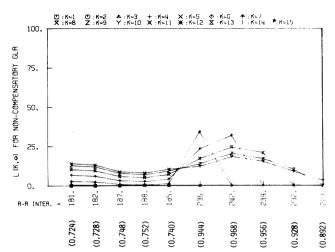


Fig. 3. Non-Compensatory Likelihood for Rhythm Jump. L(k, 6) decreases monotonically with k for $k \ge 6$, indicating that a noncompensatory beat did not occur.

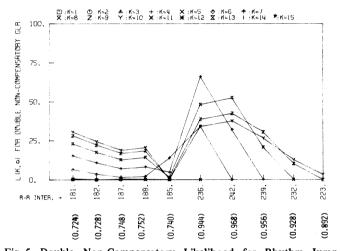


Fig. 5. Double Non-Compensatory Likelihood for Rhythm Jump. L(k, 6) increases from k = 6 to k = 7 and then decreases monotonically, indicating that double non-compensatory rhythm did not occur.

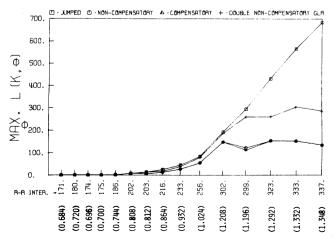


Fig. 7. Maximum Likelihoods for Gradual Slowing (Actual Data). This class may be detected by rate of increase in jump likelihood (compare Fig. 6).

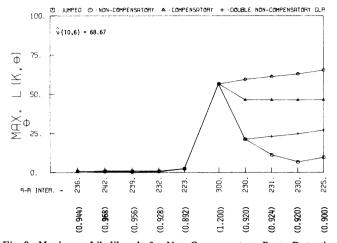


Fig. 8. Maximum Likelihoods for Non-Compensatory Beat. Detection is made at k = 7.

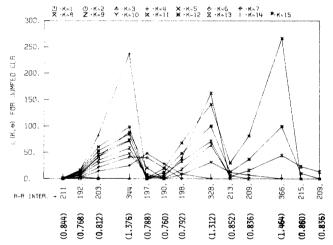


Fig. 10. Jump Likelihoods for Multiple Second Degree Blocks (Actual Data). L(k, 4), L(k, 8) and L(k, 11) decrease monotonically after initial rise, indicating no jumps have occurred.

Note the increasing likelihood of a jump as k increases, as compared to the leveling off seen for the pure jump of Figure 6. This is indicative of a persistently *changing* rhythm.

The next test was made by artificially lengthening one R-R interval in an otherwise normal sinus rhythm strip. The results are shown in Figure 8 and indicate strong identification of the non-compensatory beat with an accurate jump estimate.

The next test was made using an actual rhythm strip which included a single, interpolated atrial premature contraction close to the middle of the normal R-R interval. This corresponds to the double non-compensatory model. A summary plot is given in Figure 9 and shows that the transient rhythm is correctly identified.

The results presented thus far have involved a single transient event. It was decided to test the GLR system by using data in which multiple transients occurred within the window.³ The

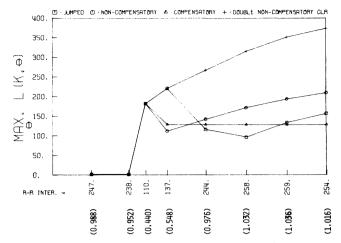


Fig. 9. Maximum Likelihoods for Interpolated Beat at k = 3. Detection is made at k = 5.

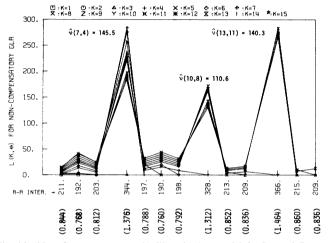
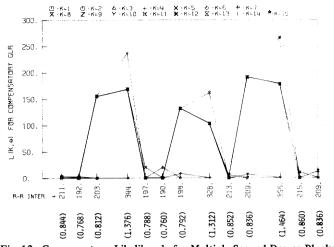


Fig. 11. Non-Compensatory Likelihoods for Multiple Second Degree Blocks (Actual Data). These are identified correctly since L(k, 4), L(k, 8) and L(k, 11) increase monotonically until next arrhythmia occurs, and then decrease only slightly.

results of a typical test on actual data are shown in Figures 10-13, in which second degree A-V block of the Wenckebach type appears. The lengthening PR intervals are not detected since we have not as yet included *P*-waves in the analysis. However, the three dropped beats, which manifest themselves as non-compensatory in nature, are easily detected (Figure 11). Note that the jump estimates, which are less in magnitude than the underlying normal sinus rhythm, would suggest that the *PR* interval prior to the dropped beat is longer than the one following it, as is the case with Wenckebach phenomenon. Note also from Figures 10-13 that the GLR detectors for all events respond to the non-compensatory beats; however, the largest, most consistent values are obtained by the proper detector.

The experimental results up to this point have been obtained using all the data and without the initialization technique of Section III. It was found by experiment that a transient rhythm near the beginning of the data resulted in poor performance. An example of this is shown in Figures 14-15 in which $L(k, \theta)$ is plotted for the first five intervals for all four categories. The first *R*-wave was a PVC, which was difficult to

³ In this paper, the data window includes all the data $(\theta \in \{1, \dots, k\})$, since the rhythm strips were of short duration. In normal operation, a fixed window length must be used. It should be noted that shorter windows are contained in the longer ones. Hence, from these results we can determine the effects of using windows of various lengths. We will comment on this in the conclusions.



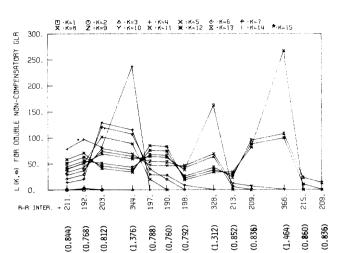
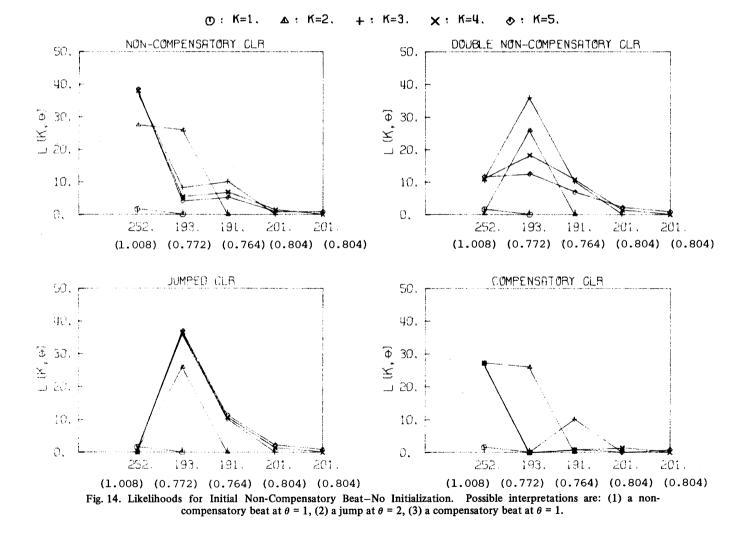


Fig. 12. Compensatory Likelihoods for Multiple Second Degree Blocks (Actual Data). Likelihoods decrease after initial rise, indicating that no compensatory beats occurred.

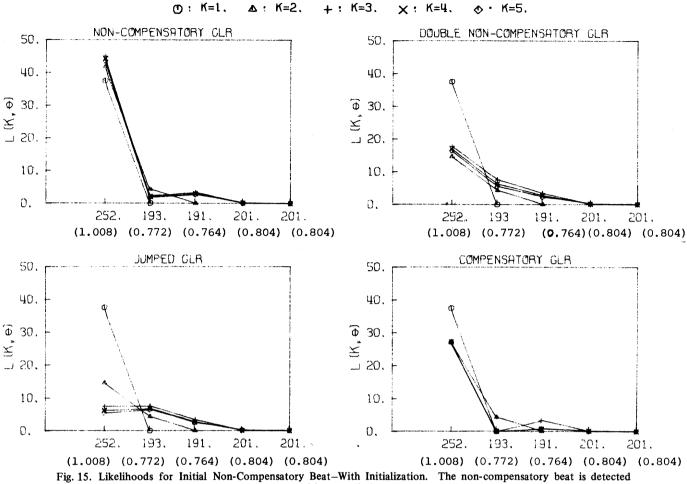
Fig. 13. Double Non-Compensatory Likelihoods for Multiple Second Degree Blocks (Actual Data). This arrhythmia is not indicated since the likelihoods decrease after the initial rise.



identify, as shown in Figure 14, with no initialization. The low value of L(1, 1) is due to the large initial error variance associated with the initial *R-R* interval estimate of 200. With proper initialization, as shown in Figure 15, the value of L(1, 1) is much higher and there is no confusion as to the existence of a non-compensatory beat at $\theta = 1$.

V. CONCLUSION

In this paper, we have developed a systematic approach to the problem of cardiac transient rhythm detection and identification using R-R intervals. The R-R intervals are modeled as being composed of two terms: (1) an underlying normal sinus rhythm, and (2) an unknown and unpredictable transient. The



without confusion.

normal sinus rhythm is estimated by a Kalman filter. The filter residuals are then monitored to detect and identify the transient rhythms. Identification is performed by comparing the residual sequence to a set of "signatures," or templates, one for each category modeled. Maximum likelihood detection is performed by correlation of the data to these "signatures." Simultaneously, an estimate of the value of the transient is obtained, which gives additional diagnostic information.

The technique can be easily mechanized for on-line operation and uses prestored gains for all computations. A series of experimental tests have been made on actual data using four phenomenological models: (1) rhythm jump, (2) noncompensatory beat, (3) compensatory beat, (4) double noncompensatory beat. The results of these tests indicate that very accurate detection and identification can be obtained. In addition, multiple transients within the window used for computations can be accurately identified.

One note of caution must be added. As indicated in Section IV, the GLR system was able to correctly determine each transient event. However, when one or more events occur, all of the detectors respond in some fashion, and one must be careful in devising decision logic that must examine a collection of likelihood ratios. As the window becomes wider (increasing the likelihood of multiple events within the window) the logic can become rather complex (how many events were there? when did they occur? what were they?). In preliminary work being performed at the present time, we have found that a window length of two (i.e., we examine and only compute $P_i(k, k)$ and $P_i(k, k-1)$) leads to accurate detection, and complete separation of successive events, with essentially no problem of confusion among the various transient rhythm diagnoses. With this system the decision logic is extremely simple, and one can devise straightforward rules for identifying events that persist over a time period longer than two intervals (e.g., jumps) by keeping track of consecutive decisions (e.g., in the jump case, we will detect a sequence of double non-compensatories over successive windows of length two). These results will be presented in a future paper.

This technique appears to be a very powerful approach to transient rhythm detection for several reasons. First, the approach is statistical and multivariable, allowing complex correlations and uncertainties to be accounted for rather easily. The cardiologist's knowledge can be used to set the likelihood thresholds, based on his overall diagnosis, rather than setting several, possibly interacting thresholds, on a lower level. Second, the approach is phenomenological, simplifying the modeling process. Categories are easily added or deleted, as desired. Third, the method appears to be accurate and relatively insensitive to modeling errors as indicated by our experimental results. Finally, the mechanization is relatively simple, and the critical gains can be computed a priori on the basis of the dynamical models and predicted uncertainty level.

Clearly the techniques for identifying transient and persistent rhythms must be combined into an overall system. This involves the inclusion of further system logic, and a prototype system along these lines has recently been developed. The phototype will be described in a future paper.

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Malcolm C. Lancaster, for a photograph and biography, see page 352 of this TRANSACTIONS.

John H. Triebwasser, for a photograph and biography, see page 353 of this TRANSACTIONS.

An Adaptive Coherent Optical Processor for Cell Recognition and Counting

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Abstract-An adaptive coherent optical processor, using discrete sampling of the Fourier spectrum, is described. The discrete sampling method was used to realize a system for recognizing and counting reticulated red blood cells using conventional blood film slides as the input field. This process is based on the assumption that a family of cells of distinct morphology will have a unique Fourier spectrum and that the intensity of the spectrum from the individual cells is additive when the population is large, randomly located, and nonoverlapping. Intensity measurements made at discrete spatial frequencies provide information on the number of cells of each type present. Inference of the counts is made through a linear estimation model obtained by a least squares regression of the intensity measurements against a set of known counts. If applied to the class of spectra for which the system model is designed, the estimation error is equal to or better than equivalent hand counting methods and is of the same order as that inherent in any least squares regression process.

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INTRODUCTION

UTOMATION of the acquisition and interpretation of A data in cytology [1], and microscopy [2] in general, has been a focus of biomedical research for nearly two decades [3, 4]. Several approaches have been employed in the pursuit of automating the techniques. Ingram and Preston [5,6] and Megla [7] have successfully applied microscopic scanning techniques and digital pattern recognition to the differential leukocyte count. Groner [8], Melamed, et al. [9], Kamentsky and Melamed [10], and Loken, Sweet, and Herzenberg [11, 12] have applied optical absorption, fluorescence, and scattering properties of cells in continuous flow systems to successfully obtain separation of cells and cell classes according to these properties. The difficulty with the present digital computer systems [13] appears to be the relatively slow rate of analysis and the expense of processing. Although the continuous flow systems can analyze up to several thousand cells per second [12] with a high degree of repeatability, the classification obtained depends to a large degree on indirect properties of the cell, such as reactions to chemical processes, and not the specific cell morphology [1]. Cost is also becoming a factor

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