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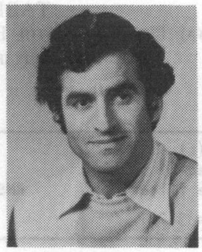

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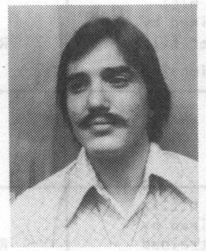

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ECG/VCG Rhythm Diagnosis Using Statistical Signal Analysis—I. Identification of Persistent Rhythms

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***Abstract*–In this paper we describe a technique for detection and classification of cardiac arrhythmias for ECG or VCG data. The approach is based on the use of *R-R* interval data and the development of simple models that describe the sequential behavior of such intervals characteristic of different arrhythmias which persist over a period of about six or more heartbeats. In the second part of this two-paper series, we will deal with arrhythmias that manifest themselves as abrupt changes in the observed *R-R* intervals.**

The techniques used to analyze observed *R-R* intervals are statistical in nature, allowing one to assign likelihoods or probabilities to various potential diagnoses. We feel that this approach is potentially quite useful in the design of detection algorithms containing relatively simple decision logic.

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I. Introduction

In this paper we consider the problems of automated rhythm analysis of electrocardiograms (ECG's) and vectorcardiograms (VCG's)–i.e., the analysis of the sequential behaviour of cardiac events as measured by the ECG or VCG [1]. A great amount of work has been done in the past in rhythm analysis, resulting in several well-known programs (e.g., [2–5]). The importance of arrhythmia detection is underlined by the increasing recognition of the role of arrhythmias as a cause of sudden death [6]. However, even in the light of the importance of and the amount of work done on the problem, it is fair to say that in practice the problem is not solved.

The key issue in detection of various arrhythmic events is the recognition of certain temporal patterns in the ECG or VCG. Pattern recognition in present programs is generally done using a hierarchical computer logic structure and extrap-

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olating for missing data. These tests are deterministic, in that specific thresholds are set for the various tests, rather than statistical, wherein probabilistic statements are given based on statistical models of the temporal pattern.

Our approach is based on the use of several statistical techniques for sequential detection and hypothesis testing. In order to apply such techniques, one must develop dynamic models describing various arrhythmic events. An attempt to develop such models based on the complex and nonlinear mechanical, chemical, and electrical mechanisms of cardiac activity would be extremely formidable and in all probability extremely sensitive, given the likelihood of modeling errors. Our approach has been phenomenological in nature. That is, we have developed simple models that accurately describe the macroscopic behavior of certain observables—the time intervals between certain cardiac events, specifically the R-waves [1]. We have chosen R-waves for our first efforts for several reasons: (1) they represent the highest signal-to-noise ratio piece of information available; (2) most arrhythmias do create disturbances in the R-R interval pattern; (3) the R-wave problem provides a test-bed for studying the feasibility of our techniques, since it may be possible to extend this approach to include P-waves.

Our approach in essence involves the calculation of "sufficient statistics" for the data. Having such informative statistics as probabilities and likelihood ratios, one is in a good position to devise simple rhythm decision rules. Thus, a major advantage of this approach is the condensing of the data via statistical techniques to a manageable set of parameters; in this manner, one eliminates the need for extremely complex decision logic. Another advantage of a statistical approach is that a continuous gradation of severity can be made on the basis of probabilities and likelihoods assigned to each arrhythmia class. The results described in this paper and in the sequel [7] deal with the development of two statistical techniques for the identification of different types of arrhythmias. The integration of these methods into an overall modular ECG/VCG analysis system is the subject of ongoing research, and an overall prototype system is described in [8].

The use of R-R interval data for rhythm analysis has been suggested in [9]-[12]. Gersch *et al.* [9] and Tsui and Wong [12] modelled the interval sequence as a three-state Markov chain consisting of either long, normal, or short intervals, differing by specific amounts from the mean value of all intervals over the data. The work reported here is different from these works in the following ways:

(1) In our work, we have made a distinction between two types of arrhythmic behaviour. In this paper we will concentrate on the first type of arrhythmias, which we call persistent—i.e., where the observed R-R interval pattern possesses some regularity or repetitive property over at least 6-10 heartbeats. In [7] we will discuss in detail the second type, the class of transient rhythms, characterized by R-R interval patterns that contain abrupt changes or irregularities lasting over a period of about six beats or less. As we will see, these two classes lend themselves to somewhat different statistical analysis techniques.

(2) The estimated magnitude of persistent patterns and transient events are calculated, along with identification of the most likely pattern. This allows some measure of the severity of the arrhythmia.

(3) We do not quantize R-R intervals into a discrete set of categories (long, normal, short), but rather we allow our system to adapt to continuous variations in these quantities. In particular, the mean R-R interval is computed recursively, and our system can respond to variations in the mean due, for example, to gradual slowing of the heart rate or respiration.

(4) Most importantly, we have based our methods on simple dynamic models which accurately represent certain features in the R-R interval patterns corresponding to different arrhythmias. The models so obtained are directly in a form that is amenable to the application of powerful statistical techniques that have been used with great success in a variety of applications.

II. Detection of R-Waves

The performance of the rhythm analysis program is highly dependent on accurate determination of the presence and location of each R-wave. If R-waves are missed, arrhythmias may be indicated when none are present, resulting in program false alarms. If R-waves are detected where none are actually present, false alarms could again be generated. More seriously, if extra (ectopic) beats are not detected, there is a possibility that this arrhythmia may not be diagnosed, especially if there is no coupling to adjacent beats.

The design of the R-wave detector has taken place in two steps: (1) elimination of low frequency baseline; (2) detection and identification of R-waves on the cleaned-up signal. The baseline was removed to provide more accurate measurements of R-wave amplitude. It has been found experimentally that this step is necessary to prevent missing the R-wave during high slope portions of the baseline.

Baseline removal was performed by a low pass, zero phase shift, moving average filter. Window length was 800 ms with a sampling interval of 40 ms. This yielded a 3 dB point of 0.3 Hz. The high slope segments of the QRS complex relative to the remainder of the waveform appear to be the most reliable indicator to identify this complex. In order to have a well-defined fiducial point for the QRS complex, we use the maximum startup slope point (the maximum slope point before the maximum amplitude of the R-wave) as the fiducial point. All R-waves in the records used for this study were correctly identified and no false alarms occurred. For details concerning the R-wave detector, we refer the reader to [13, 14].

III. Categorization and Modeling of Persistent Rhythms

In this section we develop the basic modeling philosophy used and describe the four persistent rhythm classes used in our tests. The models we have used are extremely simple and phenomenological in nature. We feel that this is of interest not only because they lead to classification algorithms that account for variations present in real data but also because the modeling philosophy is easily adapted to consider a wide variety of arrhythmias (see the transient rhythm models in Part II). The categorization of rhythms we consider is based solely on R-R intervals. Since many arrhythmias lead to other disturbances—e.g., in the contours of the P-waves or QRS complexes, in irregularities in P-P and P-R intervals—the classes described in this section cannot be used to identify uniquely every persistent rhythm. Rather, all arrhythmic patterns possessing the same R-R interval characteristics will be placed in the same category. Since most arrhythmias do manifest themselves in some way by altering the R-R interval pattern, our categories can be used to detect most arrhythmias. Further identification of the rhythm will require the incorporation of other information from the ECG or VCG. This extension is under present investigation.

The models we seek to develop in this section are of the form

$$x(k) = \Phi x(k-1) \tag{3.1}$$

$$y(k) = Hx(k) + v(k) \tag{3.2}$$

where $y(k)$ denotes the actually observed kth R-R interval and $x(k)$ is the n-dimensional pattern state vector defined such that $Hx(k)$ represents the ideal kth R-R interval. The additive "noise" term $v(k)$ represents the deviations from the ideal R-R pattern arising from two sources:

(i) the unavoidable errors in computing the R-R intervals, caused by inaccuracies in locating the fiducial points;

(ii) variations due to the fact that actual rhythms are never "textbook perfect" —i.e., even in the most regular normal pattern, the R-R intervals are not exactly the same—rather there are small, apparently random variations about the ideal underlying pattern.

We model $v(k)$ as an uncorrelated sequence of zero-mean random variables with variance R. The $n \times n$ matrix Φ models the periodicity of the particular persistent rhythm and H is a row vector. As we will see, each of these can be modeled by (3.1)-(3.2) with particular choices for Φ, H, and R.

Small Variation: This is the category for R-R intervals which exhibit small but random deviations from their mean value. This class includes normal sinus rhythm (60-100 beats/min), sinus tachycardia (> 100 beats/min), and sinus bradycardia (< 60 beats/min). Note that 2:1 SA block may be indistinguishable from sinus bradycardia. For this class, the ideal pattern is identically constant. This leads to the model

$$x(k) = x(k-1) \tag{3.3}$$

$$y(k) = x(k) + v(k). \tag{3.4}$$

Here we see that $\Phi = H = 1$. The value of R for this case is denoted R_s. In the following sections, in which we are interested in estimating the ideal pattern, it will become necessary to hypothesize initial conditions for the above model. We will assume that $x(0)$ is a random variable with mean $m(0)$ and variance $P(0)$.[1] The parameters $m(0)$, $P(0)$ and R_s are the three free parameters in the above model. The determination of reasonable values for them is discussed in subsequent sections.

Large Variation: This class is characterized by a large but random variation in the R-R interval sequence from the mean value. This class includes sinus arrhythmia and atrial fibrillation, among others. The mathematical model for large variation is identical in form to that for small variation—i.e., we use equations (3.3) and (3.4). The only difference is that the variance, R_l of $v(k)$ in the large variation case is chosen to be substantially larger than R_s.

Period-Two Oscillator (P2): This class is characterized by R-R intervals which are alternately long and short. Possible causes for this rhythm pattern are a premature impulse following very normal impulse (bigeminy) or the presence of an AV block of every third atrial impulse. A second order model which describes this oscillating rhythm pattern is

$$x(k) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} x(k-1) \tag{3.5}$$

$$y(k) = [1 \quad 0]\, x(k) + v(k) \tag{3.6}$$

where $x(k) = [x_1(k),\ x_2(k)]'$ is a two-dimensional vector. The initial state $x(0)$ is a two-dimensional random vector with a given mean $m(0)$ and covariance $P(0)$, and the noise $v(k)$ is again a white Gaussian sequence with variance R_2. Again, $P(0)$, $m(0)$, and R_2 are free parameters to be specified. Note that, ignoring the noise, $y(k)$ alternates between $x_1(0)$ and $x_2(0)$, which represent the nominal long and short intervals. Also note that, assuming we do not know a priori if the first beat is long or short, we cannot be certain whether it is $x_1(0)$ or $x_2(0)$ that represents the long beat. This is reflected in a choice of $m_1(0) = m_2(0)$ and $P_{11}(0) = P_{22}(0)$.

Period-Three Oscillator (P3): This class is characterized by an R-R interval sequence which repeats over a period of three beats. Causes for such a pattern include the following cardiac disturbances: a premature impulse that regularly follows every two normal heart beats (trigeminy); two consecutive premature impulses following a normal heart beat; a complete AV block of every fourth impulse. A model which has the desired periodicity is

$$x(k) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x(k-1) \tag{3.7}$$

$$y(k) = [1 \quad 0 \quad 0]\, x(k) + v(k) \tag{3.8}$$

where $x(k)$ is a three-dimensional vector, and $v(k)$ has variance R_3. Again, we assume $x(0)$ has mean $m(0)$ and covariance $P(0)$, and we see that the components of x represent one period of the ideal periodic sequence of R-R intervals. As in the $P2$ case, if we do not know where the pattern begins, we should choose $m_1(0) = m_2(0) = m_3(0)$ and $P_{11}(0) = P_{22}(0) = P_{33}(0)$.

IV. The Kalman Filter

The basis for our approach to the classification of persistent rhythms is the design of a set of systems that "track" the four

[1] Throughout this development we will assume that all random quantities have Gaussian distributions. This is not true physically, since Gaussian variables can be negative, while all our intervals are positive. However, this assumption is simply a mathematical convenience that has been used often in order to facilitate the development of algorithms. As we will see, our results will justify the use of this assumption.

persistent rhythm categories. Each category has been described by a model of the form (3.1), (3.2) where $x(k)$ is an n-vector and v is a zero-mean Gaussian white noise process with variance R. Also, we assume that $x(0)$ is a Gaussian random variable with mean $m(0)$ and covariance $P(0)$.

For a given model of the form (3.1), (3.2), the Kalman filter is the dynamic system that takes y as its input and produces the best (minimum-variance) estimate of x given these data. We refer the reader to [15] for the derivation of the following equations for the filter. Let $\hat{x}(j/i)$ denote the best estimate of $x(j)$ based on the data $y(1), \cdots, y(i)$. Then we have

$$\hat{x}(k/k-1) = \Phi\hat{x}(k-1/k-1) \tag{4.1}$$

$$\gamma(k) = y(k) - H\hat{x}(k/k-1) \tag{4.2}$$

$$\hat{x}(k/k) = \hat{x}(k/k-1) + M(k)\gamma(k). \tag{4.3}$$

The filter is initialized by setting $\hat{x}(0/0) = m(0)$. The gain matrix $M(k)$ is calculated from the precomputable equations

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi' \tag{4.4}$$

$$V(k) = HP(k/k-1)H' + R \tag{4.5}$$

$$M(k) = P(k/k-1)H'V^{-1}(k) \tag{4.6}$$

$$P(k/k) = P(k/k-1) - M(k)HP(k/k-1) \tag{4.7}$$

where $P(j/i)$ is the covariance of the estimate error, $x(j) - \hat{x}(j/i)$. We initialize the covariance with $P(0/0) = P(0)$.

Thus the design of the filter is determined once the a priori data $m(0)$, $P(0)$, and R are specified. These may be obtained by extensive statistical analysis of R-R interval data for each rhythm category (see [13, 14] for the description of statistical tests of this type), or they can be viewed as design parameters to be set in order to achieve good overall performance. A combination of these methods was used in this study.

Finally, note that equation (4.3) consists solely of the incorporation of the latest measurement $y(k)$ into the estimate. Note that the increment in $\hat{x}$ due to this measurement is proportional to the measurement innovation $\gamma(k)$, which from (4.2) represents the error in predicting $y(k)$ based on the best estimate of $x(k)$ using data up to but not including $y(k)$. If the model (3.1), (3.2) accurately represents the physical system that produces the observed data $\{y(k)\}$, then the $\gamma(k)$ form an uncorrelated, zero-mean sequence with covariance $V(k)$. If the model does not accurately represent the data, the $\gamma(k)$ will not have this statistical description. This fact is of central importance in the classification algorithm described in the next section.

V. The Multiple Model Technique

In the setting we have established, the persistent rhythm classification problem can be formulated as follows. We are given an observed sequence of R-R intervals $y(1)$, $y(2), \cdots$, and we hypothesize that $y(k)$ comes from one of a finite set of dynamical systems.

$$x_i(k) = \Phi_i x_i(k-1) \tag{5.1}$$

$$y(k) = H_i x_i(k) + v_i(k) \tag{5.2}$$

$$E(v_i^2(k)) = R_i, E(x_i(0)) = m_i(0) \tag{5.3}$$

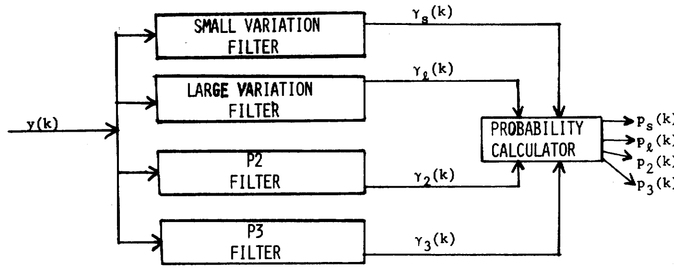

Fig. 1. Multiple Model Rhythm Identification System.

$$E((x_i(0) - m_i(0))(x_i(0) - m_i(0))') = P_i(0)$$

where $i \in (1, \cdots, N)$. We want to determine which model represents the data most closely. That is we wish to compute for each i the quantity

$p_j(k)$ = Probability, given observed data $y(1), \cdots y(k)$, that the correct model is given by (5.1) through (5.3) with $i = j$.

Having these quantities we can easily devise a decision rule for choosing the most likely model.

Following [16, 17], the calculation of the $p_i(k)$ proceeds in the following sequential manner. We implement N Kalman filters operating on the observed data—one based on each of the hypothesized models. From these filters we obtain the residual sequences $\gamma_i(k)$, $i = 1, \cdots, N$. Then we can update the probabilities via

$$p_i(k) = \frac{N(\gamma_i(k), V_i(k))\, p_i(k-1)}{\sum_{j=1}^{N} N(\gamma_j(k), V_j(k))\, p_j(k-1)} \tag{5.4}$$

where $N(\alpha, P)$ is the normal density

$$N(\alpha, P) = \frac{1}{(2\pi)^{m/2}(\det P)^{1/2}} \exp\left(-\frac{1}{2}\alpha' P^{-1}\alpha\right) \tag{5.5}$$

(here y is m-dimensional). The procedure requires an initialization of the probabilities. If we have no a priori information, we choose

$$p_i(0) = \frac{1}{N} \quad i = 1, \cdots, N. \tag{5.6}$$

In our case, we have four persistent categories ($N = 4$), and the structure of the resulting persistent rhythm classifer is given in Fig. 1. Essentially what the probability calculator does is match each of the residual sequences to the expected behaviour if the corresponding model were the correct one. The result of this is the four probabilities–$p_s(k), p_l(k), p_2(k), p_3(k)$,–which can then be fed into extremely simple decision logic (e.g., declare "undetermined rhythm" if all four probabilities are less than .8 or choose the class whose probability exceeds this threshold). The multiple model classifier is an extremely simple system to implement (all of the models are of low dimension). Also, in addition to the probabilities, the output of the classifier includes estimates of the state vectors for each model, from which one can estimate heart rate.

In practice, one will want to have a system that is capable of switching among the various rhythm classes, since switches in rhythm are possible. The multiple model technique as described up to now is very good at locking onto the correct model quickly at the beginning (see results in the next section). However, if one of the probabilities becomes very large, it is very difficult for the others to increase quickly if a rhythm change occurs (note that $p_i(k)$ is proportional to $p_i(k-1)$ in equation (5.4), and thus if $p_i(k-1)$ is very small, $p_i(k)$ will be also). Therefore, in order to improve the adaptability of the system, we have set upper and lower bounds on the probabilities. An upper bound of .97 and a lower bound of .01 have been used (with the constraint that the sum of all the probabilities is 1).

The multiple model algorithm has several parameters to be adjusted, namely the noise covariances R_s, R_l, R_2, and R_3, and the initial statistics for x–$m(0)$ and $P(0)$. Appropriate values for these quantities can be obtained by calculating sample statistics over an ensemble of R-R interval records. The values of these parameters should be chosen to yield the best performance of the identification algorithm over an ensemble of records. We have made extensive simulation tests to determine the effect of parameter choice on system performance. Some of these results are described in Section VI. A much more extensive set of results is described in [13, 14].

Note that the gains in the Kalman filters decrease monotonically with time. Thus even if a lower bound on the probabilities is set, a sudden rhythm shift may not be reflected in a corresponding change in the filter estimates. In order to overcome this difficulty, we wish to detect sudden changes in order to reinitialize the probabilities and filter gains. The design of a sophisticated detection system capable of identifying events such as compensatory prematures and interpolated beats is the subject of Part II. For the purposes of the multiple model algorithm all we need to do is implement an outlier test. The residuals of the most probable model are monitored. If the quantity $\gamma^2(k)/2V(k)$ exceeds a preset threshold (taken to be 2 in the tests described in the next section), the probabilities and filter gains are reset. Since it takes a period of time for the algorithm to lock onto the new rhythm, we do not reinitialize the outlier test until one of the probabilities has again exceeded the decision threshold of .8.

The multiple model algorithm as described to this point has difficulty distinguishing between small variation and *P2* and between small variation and *P3*. This is due to the fact that small variation is periodic with period one and hence also with period two and three. Thus the *P2* and *P3* filters can also track a small variation rhythm by estimating all of the components of $\hat{x}$ as being equal. We can improve the distinguishability by increasing the magnitudes of R_2 and R_3 relative to R_s, but we cannot increase them too much, since we will run into distinguishability problems with the large variation filter. Alternatively, we can look at the relative sizes of the components of $\hat{x}$ for *P2* and *P3*. If the rhythm is one of these, the corresponding $\hat{x}$ should have components with different magnitudes. On the other hand, if the rhythm is small variation, all of the components should be nearly equal. Thus, we can devise a rule that decreases p_2 and p_3 if their associated estimate vectors $\hat{x}_2$ and $\hat{x}_3$ have components that are too close in size. Specifically, the following algorithm has been found to work well: we do not check the components of $\hat{x}_2$ and $\hat{x}_3$ until 5 R-R intervals have been processed since initialization or reinitialization. Subsequent to this, we do the following: compute $\hat{x}_2(k/k)$ and the quantity

$$D(k) = \frac{|\hat{x}_{21}(k/k) - \hat{x}_{22}(k/k)|}{\max\,[\hat{x}_{21}(k/k), \hat{x}_{22}(k/k)]} \tag{5.7}$$

(here $\hat{x}_{21}$ and $\hat{x}_{22}$ are the two components of $\hat{x}_2$). Then in calculating the updated probabilities $p_s(k)$, $p_l(k)$, $p_2(k)$, and $p_3(k)$, we replace $p_2(k-1)$ in equation (5.4) with $p_2(k-1)$ $f(D(k))$ where

$$f(D) = \begin{cases} .2 & D \leqslant .1 \\ 4D - .2 & .1 \leqslant D \leqslant .3. \\ 1 & D \geqslant .3 \end{cases} \tag{5.8}$$

Now for p_3, compute $\hat{x}_3(k/k)$ and the quantity

$$E(k) = \frac{|\hat{x}_{31}(k/k) - \hat{x}_{32}(k/k)| + |\hat{x}_{32}(k/k) - \hat{x}_{33}(k/k)| + |\hat{x}_{33}(k/k) - \hat{x}_{31}(k/k)|}{\max\,[\hat{x}_{31}(k/k), \hat{x}_{32}(k/k), \hat{x}_{33}(k/k)]}. \tag{5.9}$$

In calculating the $p(k)$ from (5.4), we then replace $p_3(k-1)$ with $p_3(k-1)g(E(k))$ where

$$g(E) = \begin{cases} .2 & E \leqslant .5 \\ \frac{8}{3}E - \frac{17}{15} & .5 \leqslant E \leqslant .8. \\ 1 & E \geqslant .8 \end{cases} \tag{5.10}$$

We have found experimentally that these choices for f and g yield excellent system performance. Clearly other choices are possible.

VI. Experimental Results

The multiple model rhythm identification system described in the previous section has been tested on an extensive data base which included both idealized and actual R-R interval data. A total of 12 representative persistent rhythm records using actual data were selected for study. In this section, the most significant and representative results obtained with actual data are presented. More extensive results are available in [13, 14].

The results given here are for the five different cases described in Table I. Only the first twenty R-R intervals, found from lead V_5, were used, with the means and standard deviations given in Table I. The R-waves were detected using the technique described in Section II, and all R-waves were correctly identified.

The numerical results are given in Figures 2-12. In the figures, the abscissa is time, and the locations of the R-waves are given by the short vertical lines. The values of the R-R intervals are given along the abscissa, in units of samples (at 4 ms/sample) and in seconds. The a posteriori probabilities

TABLE I
ACTUAL DATA USED IN TESTING MULTIPLE MODEL HYPOTHESIS SYSTEM

Description	Mean	Standard Deviation
Small Variation 1	181.9*	4.48
Small Variation 2	234.0	7.59
Large Variation	116.1	17.09
Bigeminy (P2)	204.4	74.07
Trigeminy (P3)	141.6	47.72

*Units are samples at 4 ms/sample.

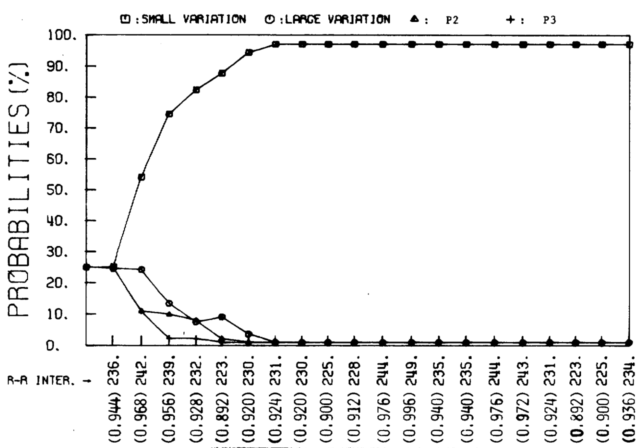

Fig. 2. A Posteriori Probabilities for Small Variation 1.

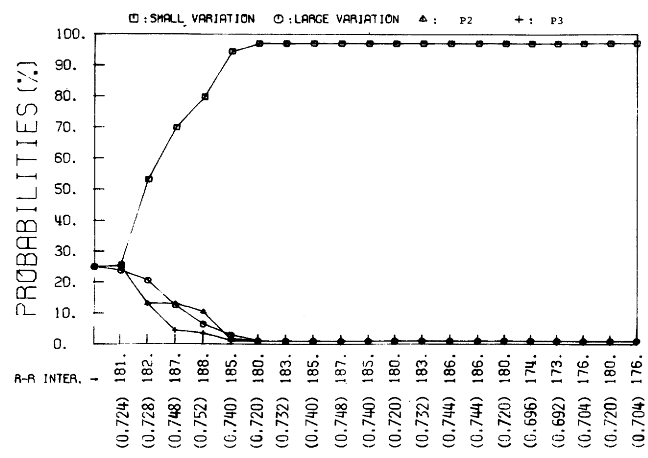

Fig. 3. A Posteriori Probabilities for Small Variation 2.

for each of the four classes are plotted along the ordinate. The lines joining the probabilities at the discrete times are used only to facilitate ease of visual interpretation, since the probabilities are defined only at the sample times, where the R-waves are located.

A large number of tests were made using different values of the filter parameters. The following set was found to give best overall results:

$$m(0) = 200, R_s = 64, R_l = 400, R_2 = R_3 = 100.$$

The initial covariance matrix was diagonal with all diagonal elements equal to 1600. The initial class probabilities were all set to 0.25.

The mean value $m(0) = 200$ is based on an average heart rate of 75 beats/min which, without any a priori information, is a reasonable estimate. Another approach to determination of $m(0)$ is to use a smoothed estimate assuming no a priori information. For example, $m(0)$ could be selected as the mean of the first few R-R intervals, or the mean of the first two R-R intervals sufficiently close together. These estimates would be noncausal but easily computed. Studies are presently under way to evaluate these approaches.

The identification performances obtained for normal sinus rhythms are shown in Figs. 2 and 3. After only five R-R intervals are processed, the probability for "small variation" has risen to about 90% and monotonically increases thereafter to

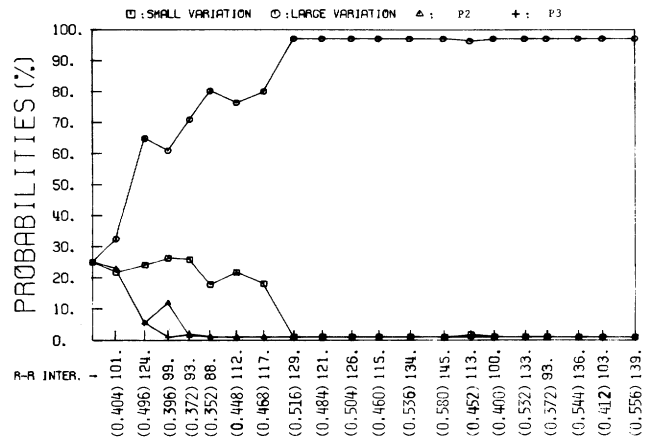

Fig. 4. A Posteriori Probabilities for Large Variation.

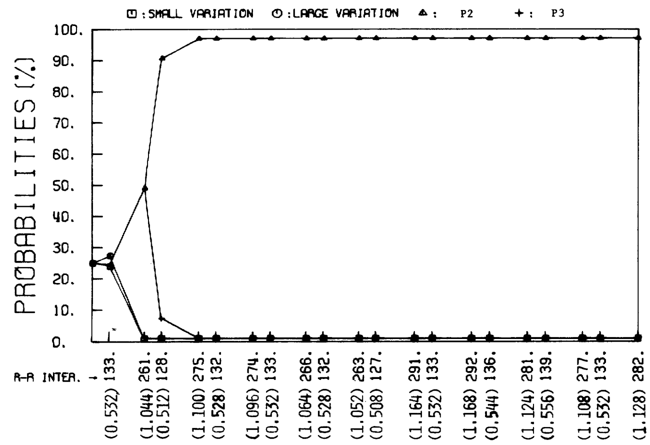

Fig. 5. A Posteriori Probabilities for $P2$.

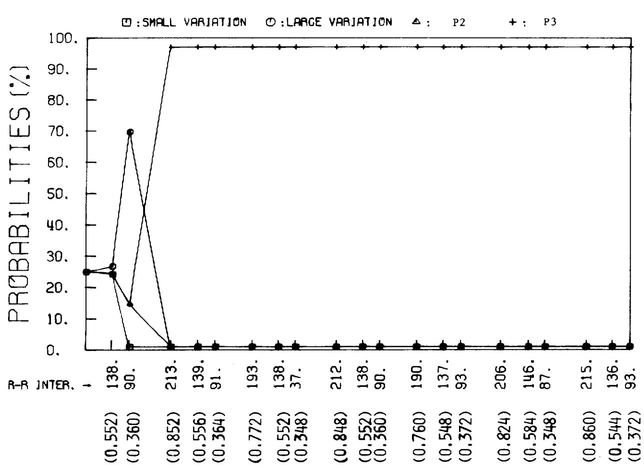

Fig. 6. A Posteriori Probabilities for $P3$.

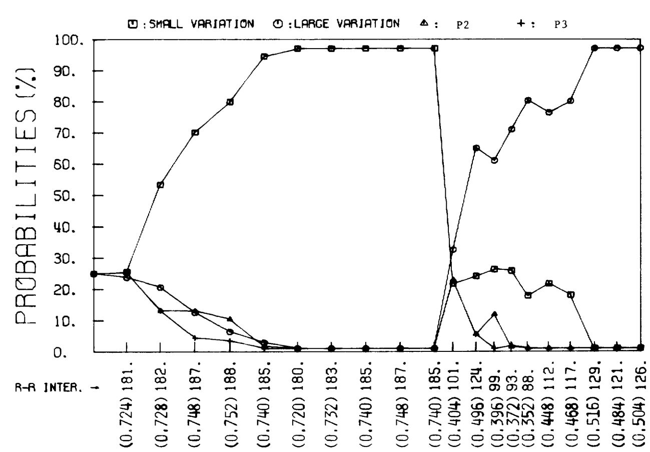

Fig. 7. A Posteriori Probabilities for Small Variation 1 → Large Variation at Eleventh R-R Interval.

the upper limit of 97%. This performance was judged to be adequate.

The performance obtained for a rhythm strip with large, essentially random sinus rhythm variation is shown in Fig. 4. The data actually corresponded to atrial fibrillation, with an average heart rate of about 140 beats/min. The "large variation" probability exceeds 90% after the 8th R-R interval has been processed. The reason accurate identification was not obtained sooner than this was the presence of the pattern $(\cdots, 99, 93, 98, \cdots)$ which is more indicative of a small variation rhythm (although it is indicative of tachycardia). Note that during this period the normal rhythm probability is increasing slightly, as would be expected. The increase in "large variation" probability at R-$R = 99$ is due to the decrease in $P2$ probability.

The performance obtained for the bigeminal rhythm strip is given in Fig. 5 and shows that the $P2$ probability rises to 90% after only three intervals have been processed, which is intuituively the smallest number necessary to provide unambiguous identification. The results for the trigeminal rhythm are shown in Fig. 6.

The results in Figs. 2-6 were obtained for pure persistent rhythms. It is of interest to assess the filter performance when there are sudden changes in persistent rhythm to assure that the response time is not degraded. Several tests were made in which two persistent rhythm strips were joined together. Although somewhat artificial, this procedure does allow determination of response to step changes in persistent rhythm. The results obtained for shifting from "small variation 1 to large variation" are shown in Fig. 7. The initial response is identical to Fig. 2 while the response following the rhythm shift is identical to the initial part of Fig. 4. This is due to the fact that the reinitialization sequence includes resetting both the filter parameters and the probabilities to their initial values when the shift is detected via the outlier test (see Section V for a description of the outlier test). Also, the initial R-R interval is reprocessed after filter reinitialization (noncausally) to achieve optimum filter response. The results obtained using other rhythm combinations followed the same pattern, as shown in Figs. 8-12.

VII. Conclusions

A new approach has been put forward in his paper for automated detection and identification of persistent cardiac rhythms. The R-R interval temporal patterns are modelled by low-order Markov processes designed to match the observed correlation functions of the data. Rhythm categories are selected on the basis of both diagnostic criteria and dynamic

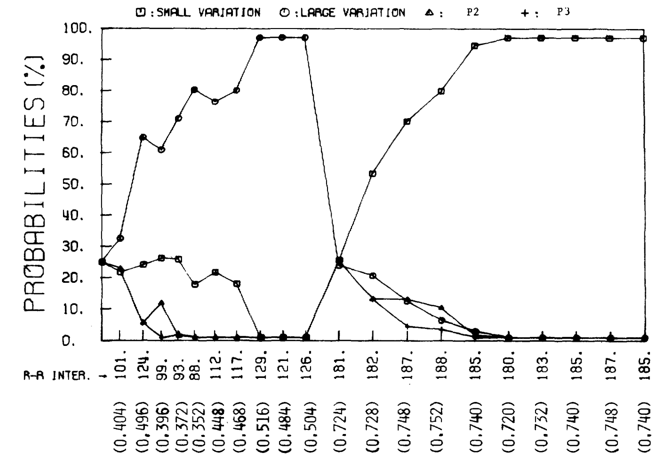

Fig. 8. A Posteriori Probabilities for Large Variation → Small Variation 2 at Eleventh R-R Interval.

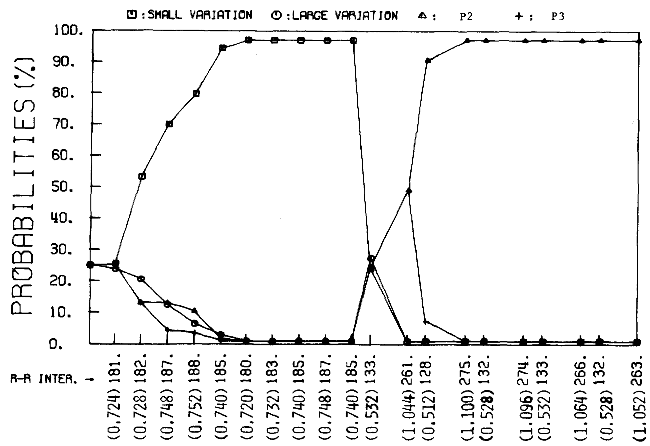

Fig. 9. A Posteriori Probabilities for Small Variation 1 → $P2$ at Eleventh R-R Interval.

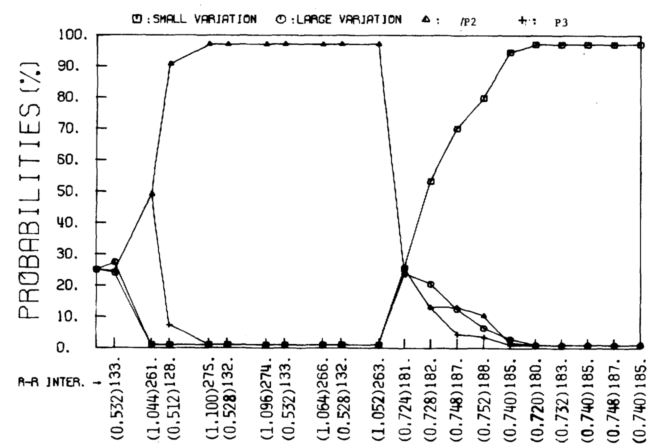

Fig. 10. A Posteriori Probabilities for $P2$ → Small Variation 1 at Eleventh R-R Interval.

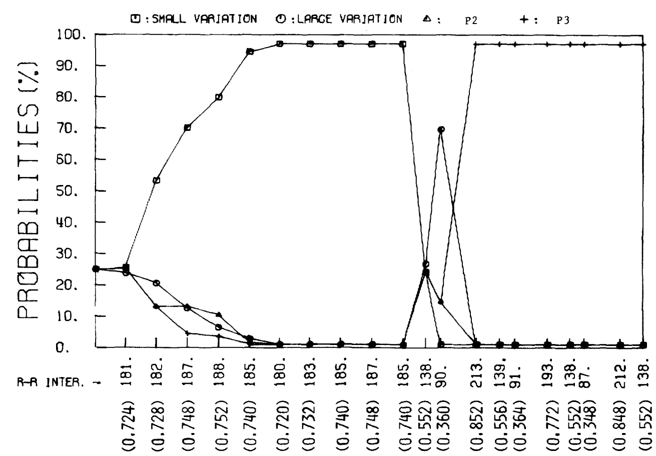

Fig. 11. A Posteriori Probabilities for Small Variation → $P3$ at Eleventh R-R Interval.

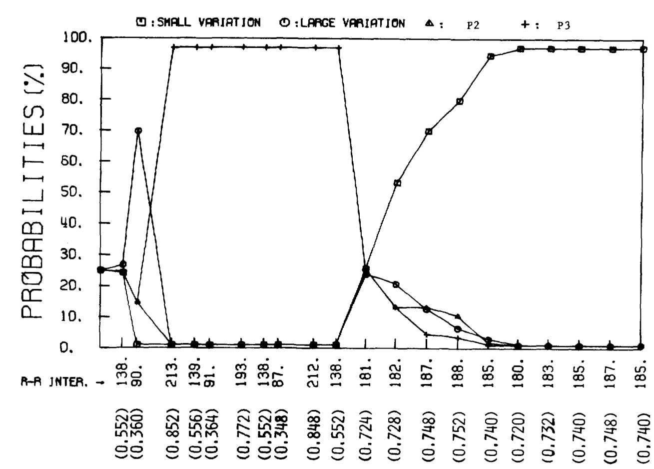

Fig. 12. A Posterior Probabilities $P3$ → Small Variation 1 at Eleventh R-R Interval.

behaviour. Additional categories are easily added if necessary. A set of tracking filters, one for each category, is designed to provide estimates of the variables associated with each category. The filter outputs are processed sequentially in a straightforward manner to obtain estimates of the a posteriori probability associated with each category. The approach is entirely statistical and takes into account fiducial point errors and actual variations within each class, as well as providing a measure of the likelihood of each rhythm category, given the data.

Testing of the method has been performed on actual rhythm data. The preliminary conclusion is that the method provides rapid and accurate identification of persistent rhythms. The method is also appealing in that it is based on phenomenological models and does not have to be extensively "tuned" with the aid of a training set. Hence, one may avoid the sensitivity problems that often arise in methods that require extensive use of a training set in order to set thresholds.

In the second part [7] we develop another dynamic model-based statistical technique for the detection and identification of transient arrhythmias.

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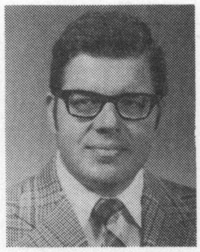

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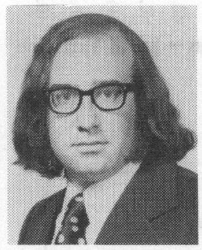

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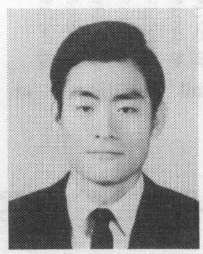

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ECG/VCG Rhythm Diagnosis Using Statistical Signal Analysis—II. Identification of Transient Rhythms

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Abstract—**The problem of detection and identification of cardiac transient rhythms, using the associated *R-R* interval sequence, is studied. A generalized likelihood ratio technique is proposed, in which the transient rhythm category is identified by means of a maximum-likelihood hypothesis test. Simultaneously, the magnitude of the change in the *R-R* interval pattern is estimated. The method is easily mechanized on-line using a moving window of data and prestored gains. Experimental results using actual data are presented to indicate the utility of the method.**

I. Introduction

This paper is the second of a two-part series on the development of an automated technique for cardiac arrhythmia detection and identification. In Part I [1] the motivation and background for this study were given, and a multiple model technique was developed for detection and identification of persistent rhythms, i.e., rhythms which are essentially unchanged over approximately 8–10 heartbeats. In addition, we presented results that showed that, with the aid of an outlier test, the multiple model algorithm was capable of detecting and adapting to switches between persistent rhythm patterns. Although this method does allow one to detect certain sudden changes in a rhythm pattern, its simplicity does not allow one to correctly identify many ectopic events such as compensatory prematures and interpolated beats.

In this paper we investigate the use of a Generalized Likelihood Ratio (GRL) technique [2-4] for detection and identification of transient arrhythmias; e.g., arrhythmias that persist over less than 8–10 heartbeats. The GLR approach is a practical method for detecting and classifying several types of transient events and for estimating the parameters that characterize the events (e.g., the degree of prematurity of a PVC). This approach has previously been found to give good experimental results [3] and will be shown to give excellent results in this application.

Following the methodology in Part I, our approach to modeling is phenomenological in nature; that is, the models are based on rather simple observations concerning the distinguishing characteristics of the *R-R* interval patterns corresponding to the various transient events. This produces a simple and quite reliable statistical identification procedure. The diagnostic capabilities of this technique are limited only

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