TABLE I

Normalized cumulative power spectra of nine human HPS waveforms. The percent of total power is given for three frequency ranges.

Patient	%<150 Hz	%<100 Hz	%<50 Hz
No. 1	98	93	58
2	98	94	62
3	99	98	92
4	100	99	62
5	100	99	78
6	100	100	96
7	99	98	88
8	100	100	100
9	97	95	78
Average ±SD	99±1.1	97±2.6	79±16

More information is also needed to determine the optimum lead placement, especially in regard to emphasizing specific structures within the HPS. It has been our experience that most investigators familiar with His bundle recordings expect to see recordings at the body surface similar to the intracardiac His bundle electrogram. This is a false expectation because the catheter recording is local in nature while the body surface lead integrates the HPS activity from a major portion of the conducting tissue.

Conclusion

While many useful and sophisticated computer techniques are used to analyze the ECG, most only replace a human interpreter and do not provide additional fundamental information. However, the digital processing technique described here yields the His-Purkinje ECG and quantifies a new component of the ECG. If perfected, the technique would allow further screening of patients, perhaps in lieu of cardiac catheterization, and would be more amenable to follow-up studies on patients. In addition, the noninvasive nature of the procedure enhances its application to a much larger segment of the population with conduction system disorders. The technique is still in the early stages of development and such parameters as optimum bandwidth and lead axis remain to be determined.

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A Statistical Approach to Rhythm Diagnosis of Cardiograms

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***Abstract*—A new method is presented for detection and classification of arrhythmias in electrocardiograms on the basis of *R-R* interval data. A set of phenomenological models for both persistent and transient rhythms is developed to match observed statistical variations. Arrhythmias are identified by calculating statistical probabilities and likelihoods associated with these models using two recently developed techniques. The important system design considerations are described. Finally, representative results using actual arrhythmia data are presented to illustrate the system performance.**

I. Introduction and Overview

In this paper we describe a system for the detection and classification of arrhythmias in electrocardiograms. The basis for our approach is the determination of a set of dynamic models that accurately describe the sequential behavior of the elapsed times between consecutive heartbeats ("*R-R* intervals"). Specifically, a number of arrhythmias are characterized by persistent patterns of *R-R* intervals, while others involve abrupt changes in the interval pattern. For each of these we develop a simple model that generates an interval pattern with the corresponding statistical properties. Using these models, we then apply two statistical techniques for the identification of persistent patterns and for the detection of abrupt changes.

It is this aspect that is novel in our approach to arrhythmia analysis. That is, we have developed very simple phenomenological models that accurately describe the statistical behavior of *R-R* interval patterns. Using these models we can apply powerful statistical techniques to develop an ECG analysis system that is simple and robust and whose performance can be accurately determined as a function of a very small number of design parameters.

In the next section we describe the dynamic models for various classes of arrhythmias. Section III summarizes the methods and system design considerations, while in Section IV we present some results to illustrate the performance of prototype system that has been developed. Our treatment is necessarily brief. For a complete development, we refer the reader to [1], [2].

II. Modeling of R-R Interval Patterns

Let $y(k)$ denote the actually observed kth *R-R* interval. We think of y as being the output of an *R-R pattern generator*, which is characterized by the state vector $x(k)$. The output $Hx(k)$ represents the *ideal* kth *R-R* interval, which differs from $y(k)$ by the noise $v(k)$, which arises from two sources:

i) the unavoidable errors in computing *R-R* intervals, caused by inaccuracies in locating the fiducial points.
ii) variations due to the fact that actual rhythms are never "textbook perfect", rather there are small, apparently random variations about the ideal underlying pattern.

Models are first described for several persistent rhythms.

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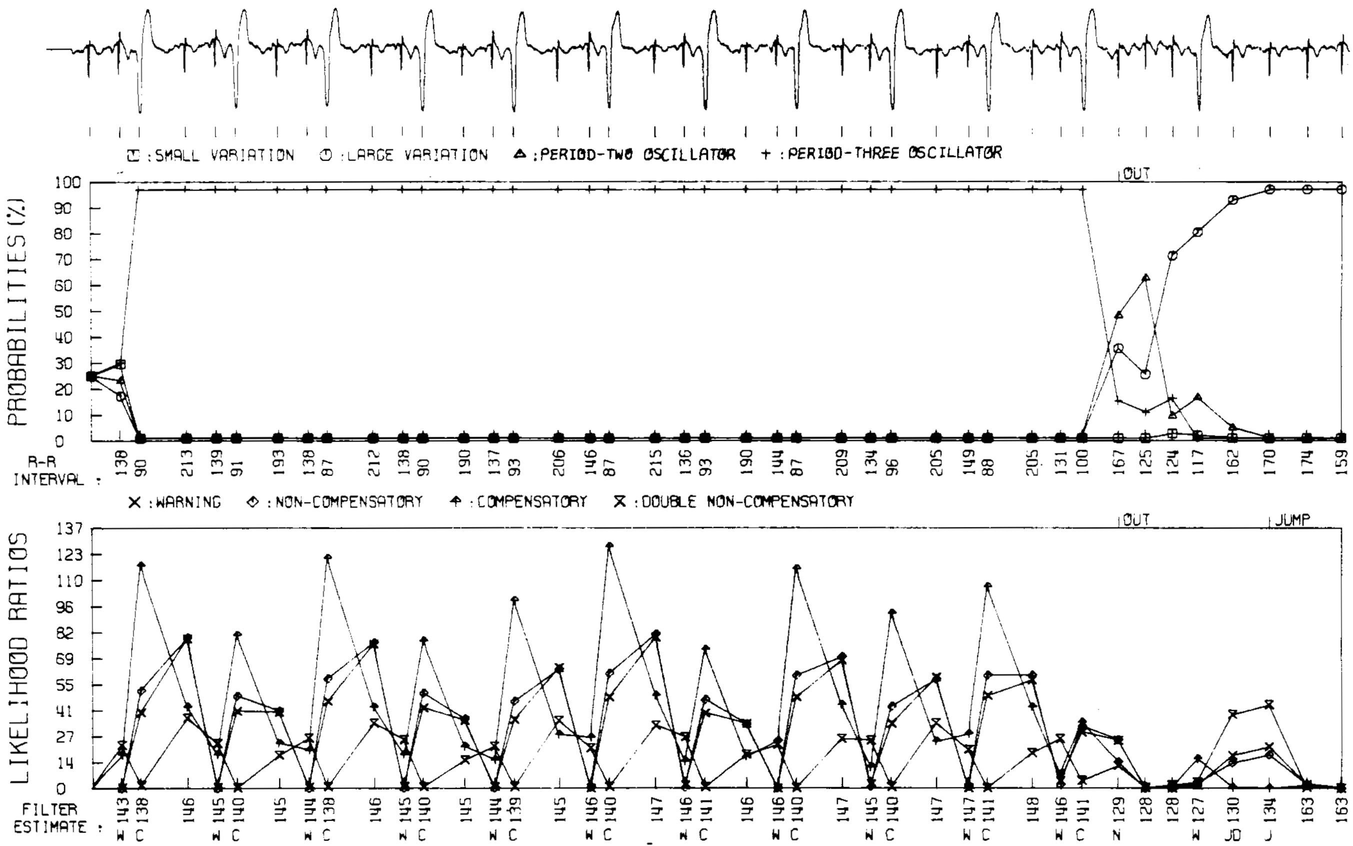

Fig. 1. Example of rhythm analysis prototype test results.

Small Variation: This class exhibits small but random deviations from the mean value of the *R-R* intervals (e.g., normal sinus rhythm)

$$x(k) = x(k-1), \qquad y(k) = x(k) + v(k) \tag{2.1}$$

where v is zero mean Gaussian and white, with variance R_s. The initial mean $m(0)$ and variance $P(0)$ of $x(0)$, as well as R_s are design parameters, reasonable values for which can be determined with the aid of the statistical techniques described in [1].

Large Variation: This class is characterized by large but random variations in the *R-R* intervals (e.g., sinus arrhythmia). The model for this class is also given by (2.1), the only difference being that the variance of v is taken to be $R_l > R_s$.

Period Two Oscillator: This class is characterized by *R-R* intervals which are alternately long and short (e.g., bigeminy)

$$x(k) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} x(k-1) \tag{2.2}$$

and

$$y(k) = (1, 0)\, x(k) + v(k) \tag{2.3}$$

where the initial mean $m(0)$ and covariance $P(0)$ of $x(0)$ and the variance R_2 of v are free parameters to be determined by some statistical means.

Period Three Oscillator: This class exhibits an *R-R* interval sequence with a period of 3 (e.g., trigeminy)

$$x(k) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x(k-1) \tag{2.4}$$

and

$$y(k) = (1 \quad 0 \quad 0)\, x(k) + v(k). \tag{2.5}$$

Again $m(0)$, $P(0)$, and R_3 (the variance of v) are free parameters.

We now turn our attention to the models of transient events. All such events are modeled as sudden, unpredictable changes on an otherwise normal record. Thus the basic model is (2.1) with covariance of $v = R_s$, and the various transient events are modeled as changes in this pattern.

Rhythm Jump: This class is characterized by a sudden change in the heart rate

$$x(k) = x(k-1) + \nu \delta_{\theta,k}. \tag{2.6}$$

Here ν is the unknown size of the shift in the average *R-R* interval at the unknown time θ. Also, δ_{ij} is the Kronecker delta ($\delta_{ij} = 0$, $i \neq j$, while $\delta_{ii} = 1$).

Noncompensatory Beat: This class is characterized by the presence of a single lengthened or shortened *R-R* interval (e.g., SA block, PAC)

$$x(k) = x(k-1) + \nu[\delta_{\theta,k} - \delta_{\theta,k-1}] \tag{2.7}$$

i.e., $x(k) = x(0)$ for $k \neq \theta$, and $x(\theta) = x(0) + \nu$.

Compensatory Beat: This class is characterized by an isolated premature *QRS* complex followed by a compensatory pause before the following beat (i.e., a PVC)

$$x(k) = x(k-1) + \nu[\delta_{\theta,k} - \delta_{\theta,k-1} - \delta_{\theta,k-2}]. \tag{2.8}$$

Double Noncompensatory Beat: This class is characterized by two consecutive shortened or lengthened *R-R* intervals

$$x(k) = x(k-1) + \nu[\delta_{\theta,k} - \delta_{\theta,k-2}]. \tag{2.9}$$

III. An Arrhythmia Detection Technique

Examining the models for the persistent rhythm classes, we see that they are all linear systems. Thus, given a sequence of observed *R-R* intervals, we can use the multiple hypothesis method [1], [3]—consisting of a bank of Kalman filters, one for each of the models—to compute the probabilities for each of the persistent rhythm categories based on analysis of the filter residuals.

In the case of the transient categories, the generalized likelihood ratio (GLR) technique [2], [4] has been implemented. This approach involves the implementation of a Kalman filter based on the small variation model. The residuals of this filter are then fed into several matched filters that compute most likely times and the likelihood ratios for each type of transient event. Estimates of the jump ν are also obtained.

The prototype system consists of both of these subsystems with several additional features designed to:

a) shorten the response time of the multi-filter in the case of a shift from one normal rhythm pattern to another one;
b) improve the distinguishability of small variation from the period two and three oscillators and from large variations;
c) speed up the GLR identification process by looking at a narrow "window" of the most recent data;
d) provide an initialization procedure for all filters to enhance detection of events at the beginning of an ECG record;
e) reset the system subsequent to the detection of a transient event.

IV. Results

Fig. 1 depicts the result of one test on real ECG data using the prototype system. The format of the figure is as follows: At the top of the figure is the actual ECG waveform being processed. The small vertical lines beneath the waveform are the R-wave detector. The multifilter probabilities (in percentage form) are displayed next, where the *R-R* intervals (between the present R-wave and the preceding one) are measured in units of 4 ms. The symbol "OUT" is used to indicate a multifilter outlier—an indication that a previously identified persistent pattern has been interrupted.

The GLR likelihood ratios are plotted below the multifilter results. Again the horizontal axis is actual time, while the numbers given represent the running estimate of the mean *R-R* interval, as produced by the small variation GLR filter. We note that in addition to the categories, "noncompensatory" (*N*), "compensatory" (*C*), and "double noncompensatory" (*D*), we have the category "warning" (*W*), which indicates that the preceding *R-R* interval is fine but that the present one is aberrant. This signal tells us that when we look at the next interval, we should be able to decide among the various transient categories. The actual GLR decisions are located beneath the plot. In addition to *W*, *C*, *N*, and *D*, two other symbols are used. "*JD*" denotes the detection of a double noncompensatory and indicates that it might really be a jump—a question that is resolved upon looking at the next interval. Finally, at the top of the GLR plot, the times at which the GLR filter is adjusted are indicated. "JUMP" indicates that the filter estimate has been adjusted following the detection of a jump. "OUT" indicates that the GLR filter has been reinitialized. This only happens if the multifilter has locked onto period two or three oscillation and an outlier has been detected.

The waveform in Fig. 1 contains a number of premature ventricular contractions (PVC). For most of the record, these fall into a trigeminal pattern. As the results indicate, the multifilter locks onto period three oscillation (*P*3) and GLR detects the PVC's as compensatory beats. When the *P*3 pattern ends, the multifilter signals an outlier which leads to a reinitialization. The reinitialization of the GLR small variation filter yields an initial estimate of 124.5 (the average of the second and third intervals following the outlier), and thus, when the first *R-R* interval (the compensatory pause = 167) is processed, the GLR system signals a noncompensatory. Following this, the *R-R* interval pattern consists of 3 short *R-R* intervals followed by 4 long ones. As one might suspect, the multifilter locks onto large variation, while GLR detects the shift from the fast rhythm to the slower one. Note that the last PVC falls as the beat dividing the series of short and long *R-R* intervals, and, looking at the sequence of intervals alone, this beat is not premature. Incorporation of other information, such as gross waveform morphology, will lead to a clearer indication that this is an ectopic beat.

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Reduced Spline Representations for EEG Signals

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Abstract: **EEG data were approximated in a least squares sense by a normalized B-spline system having a prescribed knot sequence. The frequency content of the EEG is preserved in the power spectrum of a splined EEG having substantially fewer knots than sample points.**

I. Introduction

The EEG has a special appeal to the biosignal processing and electrical engineering community, in addition to providing significant and useful electrophysiological parameters for both research and clinical investigators. The EEG is a) of relatively low frequency (0–50 Hz) content, b) contains considerable structure and noise, c) consists of 8, and often 16, channels of data, and d) frequently consists of record lengths obtained in 6–12 h sleep studies which involve millions of data points. The training of computer scientists and electrical engineers may be brought to bear on numerous facets of this rich area of activity [1].

Because of the wide interdisciplinary appeal of the EEG signal a considerable variety of analysis techniques have been proposed by statisticians, biomathematicians, computer scientists and electrical engineers. These methods have included Fourier spectral analysis via the FFT, auto- and cross-correlation methods, time series models, pattern recognition, complex demodulation, bispectrum analysis, and time-domain period analysis.

In this paper we examine a method of EEG analysis which complements certain aspects of these traditional methods. A spline technique which serves to filter the EEG is followed by Fourier spectral analysis. It is shown that with a *B*-spline basis it is possible to reduce the information content of the *N*-point EEG data set into *M* points which are the knots of the spline basis. This results in an *N*/*M* data compression which retains the low frequency information inherently present in the EEG.

The advantages of and potential utility to routine EEG signal processing resulting from a spline/Fourier transformation include a) the smoothing or filtering of the EEG data giving visually pleasing EEG structural features, b) data compression resulting in a reduced data set, and c) efficient feature extraction resulting from Fourier transformation of the reduced data set.

II. Spline Representation

Although the term "spline" now means many things to many people, we will adhere to the notion that a spline is a piecewise polynomial of a given order (degree plus one). The points at which the polynomial pieces join in a prescribed smooth fashion are called knots. In the early days of approximation, polynomials were used extensively, but it was soon discovered that polynomial approximations tended to oscillate much more and approximate less than one would like. Additionally, in order to obtain greater accuracy, the degree had to be increased, causing many numerical problems. Recent work on the use of splines in function approximation has minimized the above difficulties, as will be indicated below.

There are several methods of obtaining spline approximants to data $\{(x_i, y_i)\}_{i=1}^{L}$. Conceptually, the simplest method is via interpolation. That is, find a spline so that $s(x_i) = y_i$. It is well-known that interpolation of noisy data does not produce satisfactory approximations to the unknown function from which the data arises. The least squares approximation to the data is a time-honored method of smoothing the data and producing a reasonable approximation. This is basically the method we use.

Let a positive integer k be given and let the knot sequence t satisfy,

$$t =: t_1 = \cdots t_k = a < t_{k+1} \leqslant \cdots \leqslant t_n < t_{n+1} = \cdots = t_{n+k} = b.$$

The spline basis most often preferred by people involved in spline computations is that composed of the normalized *B*-spline [2]. The

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